Q. State & explain Fermat's Principle of stationary time and deduce (a) the law of reflection and refraction (b) the mirror formula (c) lens formula.

Fermat's Principle :→

Fermat stated that – "A ray of light travelling from one point to another by any number of reflections and refractions follows that particular path for which the time taken is least."

This principle was then called "Fermat's principle of least time." However*, there are a number of cases in which the real path of light is one for which the time taken is maximum. density, time of

Fermat's principle is stated in most general form as – "A ray of light travelling from one point to another by any number of reflections and refractions follows a path for which, compared with all other neighbouring paths, the time taken is either a minimum or a maximum."

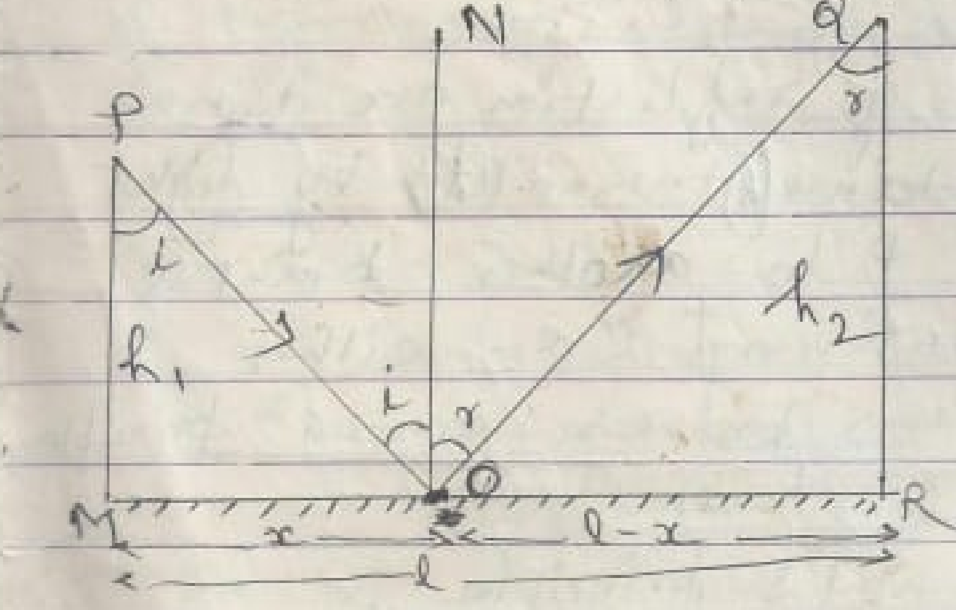
*not changing

This is known as Fermat's principle of Stationary* time." Fermat's principle may also be expressed in terms of stationary path as –

The path followed by a ray of light is moving from one point to another point after any number of reflections or refractions, would always be stationary.

Mathematically, $\int \mu\, ds$ is stationary. $\mu = c/v$, ds = is a small unit of path

Fermat's principle from the laws of Reflection :–



Fermat's principle can be established from the laws of reflection.

Let a ray of light from a point P be reflected by a plane mirror MR to another point Q. Let i be the angle of incidence and r be the angle of reflection. O is the point of incidence of light on mirror.

Let h_1 and h_2 be the lengths of the perpendiculars drawn from P and Q on the mirror and MR = l and MO = x So OR = (l – x)

$$t = \frac{\sqrt{h_1^2 + x^2}}{v_1} + \frac{\sqrt{h_2^2 + (l-x)^2}}{v_2} \quad \text{--- (1)}$$

The length of path POQ and hence time t taken for this purpose will depend on the point O i.e. on the value of x.

Differentiating t with respect to x

$$\frac{dt}{dx} = \frac{1}{2}\frac{(h_1^2 + x^2)^{-1/2}}{v_1} \cdot 2x + \frac{1}{2}\frac{[h_2^2 + (l-x)^2]^{-\frac{1}{2}} \cdot 2(l-x)(-1)}{v_2}$$

or, $$\frac{dt}{dx} = \frac{x}{v_1\sqrt{h_1^2 + x^2}} - \frac{(l-x)}{v_2\sqrt{h_2^2 + (l-x)^2}} = \frac{\sin i}{v_1} - \frac{\sin r}{v_2}$$

or, $$\frac{dt}{dx} = \frac{1}{v_0}\left\{\frac{v_0}{v_1}\sin i - \frac{v_0}{v_2}\sin r\right\}$$ where v_0 = velocity of light in vacuum

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or, $$\frac{dt}{dx} = \frac{1}{v_0}\left\{\mu_1 \sin i - \mu_2 \sin r\right\} \quad \text{--- (2)}$$

Now, from law of refraction, $\frac{\sin i}{\sin r} = \frac{\mu_2}{\mu_1}$

$\therefore \mu_1 \sin i = \mu_2 \sin r$ (Snell's law)

Putting this value in eq^n (2), we get $\frac{dt}{dx} = 0$

$\therefore t =$ constant.

Hence, if the law of refraction is taken to be true, then the time taken by light to go from P to Q is stationary, which is Fermat's principle law.

<u>Law of refraction from Fermat's principle</u>

According to Fermat's principle, $t =$ constant

$\therefore \frac{dt}{dx} = 0 \qquad \therefore$ from eq^n (2) $\mu_1 \sin i - \mu_2 \sin r = 0$

$\therefore \mu_1 \sin i = \mu_2 \sin r$

$\therefore \frac{\sin i}{\sin r} = \frac{\mu_2}{\mu_1} =$ constant

This is Snell's law for refraction of light.

$\therefore \cos\alpha + \cos\beta + \cos\gamma = ON^*\left(h^2/a^2 + \ldots\right) = 1$

or, $ON^* = \dfrac{1}{\sqrt{h^2/a^2 + k^2/b^2 + l^2/c^2}}$

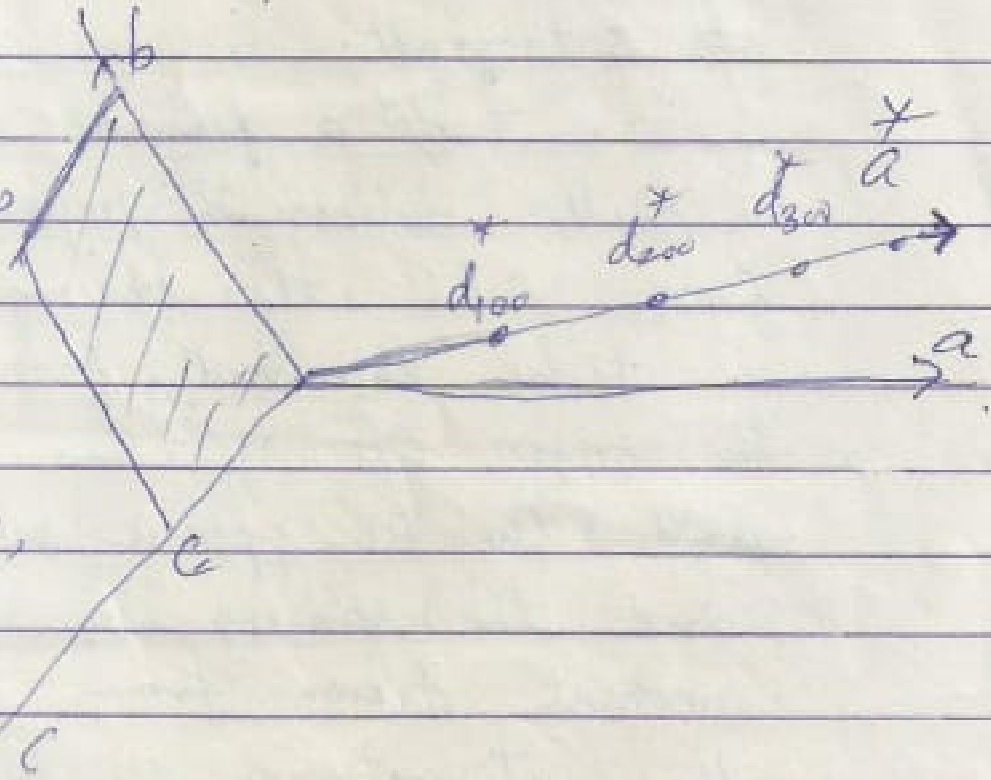
i.e. $d_{hkl} = \left(h^2/a^2 + \dfrac{k^2}{b^2} + \dfrac{l^2}{c^2}\right)^{-\frac{1}{2}}$

<u>Reciprocal lattice:</u>

A method for representing the slopes and interplaner spacings of the lattice planes has been provided by a concept known as reciprocal lattice. It is built from the direct crystal lattice by constructing normal to each plane from a common origin, the length of the normal being reciprocal of the interplanar spacing, the end-point of each such normal representing a point of reciprocal lattice. The collection of such points represents:

(1) a collection of the slopes of direct planes by the inclinations of normals and (2) a collection of interplaner spacings of the direct lattice planes in terms of reciprocal spacings.

All planes parallel to bc plane will be called (h00) planes. The spacings of such planes will be denoted by d_{h00}. Thus the spacings will be $d_{100}, d_{200}, d_{300}, d_{400}, \ldots d_{h00}$.



But $d_{200} = \frac{1}{2} d_{100}$

$d_{300} = \frac{1}{3} d_{100}$

$d_{400} = \frac{1}{4} d_{100}$

$d^*_{100} = \dfrac{1}{d_{100}}$

$d^*_{200} = 2 \ldots \dfrac{1}{d_{200}}$, $d^*_{300} = \dfrac{1}{d_{300}} = \dfrac{3}{d_{100}} = 3d^*_{100} \ldots$ $d^*_{h00} = h d^*_{100}$

But area of the base $= |\vec{b}\times\vec{c}|$, hence from eqn (1)

$$\frac{1}{d_{100}}\vec{n} = \frac{\vec{b}\times\vec{c}}{V}$$

$\therefore\ \vec{a}^* = \frac{1}{d_{100}}\vec{n} = \frac{\vec{b}\times\vec{c}}{V}$, since $\vec{a}^* = \vec{d}^*_{100}$

But $V = \vec{a}\cdot\vec{b}\times\vec{c}$

$$\therefore\ \vec{a}^* = \frac{1}{d_{100}}\vec{n} = \frac{\vec{b}\times\vec{c}}{V} = \frac{\vec{b}\times\vec{c}}{\vec{a}\cdot\vec{b}\times\vec{c}}$$

Similar expressions can be obtained for $\vec{b}^*$ and $\vec{c}^*$. These three vectors are selected as the three reciprocal axes for representing three dimensional reciprocal lattice. Thus the three reciprocal axes are defined by the following three reciprocal repeat distances:

a, b, c

$$\vec{a}^* = \frac{\vec{b}\times\vec{c}}{\vec{a}\cdot\vec{b}\times\vec{c}}$$

$$\vec{b}^* = \frac{\vec{c}\times\vec{a}}{\vec{a}\cdot\vec{b}\times\vec{c}}$$

$$\vec{c}^* = \frac{\vec{a}\times\vec{b}}{\vec{a}\cdot\vec{b}\times\vec{c}}$$

The reciprocal axes bear the following relations:

$\vec{a}^*$ is normal to $\vec{b}$ and $\vec{c}$,

$\vec{b}^*$ is normal to $\vec{c}$ and $\vec{a}$,

$\vec{c}^*$ is normal to $\vec{a}$ and $\vec{b}$.

Also, $\vec{a}^*\cdot\vec{b} = 0$ $\qquad \vec{a}^*\cdot\vec{c} = 0$

$\vec{b}^*\cdot\vec{c} = 0$ $\qquad \vec{b}^*\cdot\vec{a} = 0$

$\vec{c}^*\cdot\vec{a} = 0$ $\qquad \vec{c}^*\cdot\vec{b} = 0$

It can be shown that $\vec{a}^*\cdot\vec{a} = 1$, $\vec{b}^*\cdot\vec{b} = 1$, $\vec{c}^*\cdot\vec{c} = 1$

Thus, $\vec{\sigma}_{hkl} = h\vec{a}^* + k\vec{b}^* + l\vec{c}^*$